**1.**

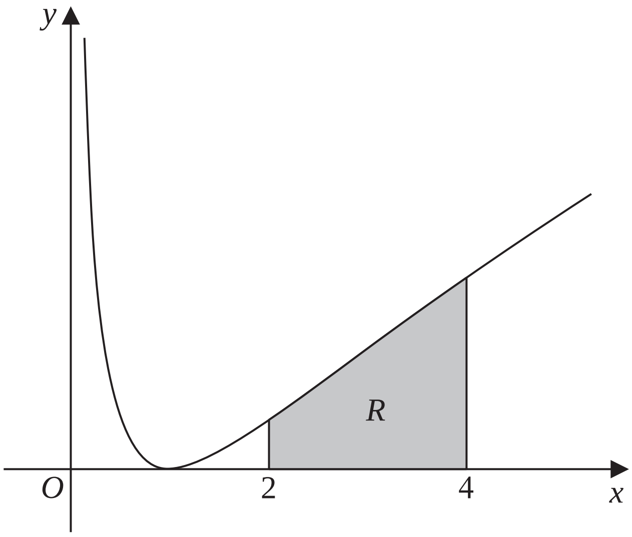


**Figure 2**

Figure 2 shows a sketch of part of the curve with equation

$$y = (\ln x)^2 \qquad x > 0$$

The finite region $R$, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the $x$-axis and the line with equation $x = 4$

The table below shows corresponding values of $x$ and $y$, with the values of $y$ given to 4 decimal places.

$4-3.5=0.5$

| $x$ | 2 | 2.5 | 3 | 3.5 | 4 |
|---|---|---|---|---|---|
| $y$ | 0.4805 | 0.8396 | 1.2069 | 1.5694 | 1.9218 |

$h=0.5$ (1)

(a) Use the trapezium rule, with all the values of $y$ in the table, to obtain an estimate for the area of $R$, giving your answer to 3 significant figures.

**(3)**

(b) Use algebraic integration to find the exact area of $R$, giving your answer in the form

$$y = a(\ln 2)^2 + b\ln 2 + c$$

where $a$, $b$ and $c$ are integers to be found.

**(5)**

a) $R = \int_2^4 (\ln x)^2 dx \approx \frac{0.5}{2}\left[0.4805 + 1.9218 + 2(0.8396 + 1.2069 + 1.5694)\right]$ (1)

$= 2.41$ (3sf) (1)

b) $R=\int_2^4 (\ln x)^2\,dx$

by parts: $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

find $\int (\ln x)^2\,dx$

let $u=(\ln x)^2$ $\quad \frac{dv}{dx}=1$

$\frac{du}{dx}=\frac{2\ln x}{x}$ $\quad v=x$ (1)

$\int (\ln x)^2\,dx = x(\ln x)^2 - 2\int \ln x\,dx$ (1)

$\int \ln x\,dx$

let $u=\ln x$ $\quad \frac{dv}{dx}=1$

$\frac{du}{dx}=\frac{1}{x}$ $\quad v=x$

$\int \ln x\,dx = x\ln x - \int 1\,dx$

$= x\ln x - x$

$\int (\ln x)^2\,dx = x(\ln x)^2 - 2(x\ln x - x)$

$= x(\ln x)^2 - 2x\ln x + 2x$ (1)

$R=\int_2^4 (\ln x)^2\,dx = \left[x(\ln x)^2 - 2x\ln x + 2x\right]_2^4$

$= 4[\ln 4]^2 - 8\ln 4 + 8 - 2(\ln 2)^2 + 4\ln 2 - 4$

$= 4[2\ln 2]^2 - 16\ln 2 - 2[\ln 2]^2 + 4\ln 2 + 4$ (1)

$= 16(\ln 2)^2 - 12\ln 2 - 2(\ln 2)^2 + 4$

$= 14(\ln 2)^2 - 12\ln 2 + 4$ (1)

$\ln 4 = \ln 2^2 = 2\ln 2$

**2.** The table below shows corresponding values of $x$ and $y$ for $y = \log_3 2x$

The values of $y$ are given to 2 decimal places as appropriate.

$h = 1.5$

| $x$ | 3 | 4.5 | 6 | 7.5 | 9 |
|---|---|---|---|---|---|
| $y$ | 1.63 | 2 | 2.26 | 2.46 | 2.63 |

(a) Using the trapezium rule with all the values of $y$ in the table, find an estimate for

$$\int_3^9 \log_3 2x \, dx$$

**(3)**

Using your answer to part (a) and making your method clear, estimate

(b) (i) $\int_3^9 \log_3 (2x)^{10} \, dx$

(ii) $\int_3^9 \log_3 18x \, dx$

**(3)**

a) $\int_3^9 \log_3 2x \, dx \doteq \frac{1.5}{2}\left(1.63 + 2.63 + 2(2 + 2.26 + 2.46)\right)$ (1) (1)

$= \frac{531}{40} = 13.3$ (1)

b) i) $\int_3^9 \log_3 (2x)^{10} \, dx = 10 \int_3^9 \log_3 (2x) \, dx$

$= 10 \times 13.3$

$= 133$ (1)

(ii) $\int_3^9 \log_3 18x \, dx = \int_3^9 \log_3 9 \times 2x \, dx$

$= \int_3^9 \log_3 9 \, dx + \int_3^9 \log_3 2x \, dx$ (1)

$= \int_3^9 2 \, dx + 13.275$

$= \left[ 2x \right]_3^9 + 13.275$

$= (2\times9) - (2\times3) + 13.3$

$= 18 - 6 + 13.3$

$= 25.3$ (1)

**3.** A continuous curve has equation $y = f(x)$.

The table shows corresponding values of $x$ and $y$ for this curve, where $a$ and $b$ are constants.

$h = 4 - 3.8$
$= 0.2$ (1)

| $x$ | 3 | 3.2 | 3.4 | 3.6 | 3.8 | 4 |
|---|---|---|---|---|---|---|
| $y$ | $a$ | 16.8 | $b$ | 20.2 | 18.7 | 13.5 |

The trapezium rule is used, with all the $y$ values in the table, to find an approximate area under the curve between $x = 3$ and $x = 4$

Given that this area is 17.59

(a) show that $a + 2b = 51$

$$\int_0^n f(x)\,dx = \frac{1}{2}h\left[(y_0 + y_n) + 2(y_1 + \ldots + y_{n-1})\right]$$

where $h$ = interval size

**(3)**

Given also that the sum of all the $y$ values in the table is 97.2

(b) find the value of $a$ and the value of $b$

**(3)**

a) $\frac{0.2}{2}\left(a + 13.5 + 2[16.8 + b + 20.2 + 18.7]\right) = 17.59$ (1)

$a + 13.5 + 2(b + 55.7) = 175.9$

$a + 2b + 111.4 + 13.5 = 175.9$

$a + 2b = 51$ (1) (1)

b) $a + 16.8 + b + 20.2 + 18.7 + 13.5 = 97.2$ (1)

$a + b = 28$ (2)

solve (1) and (2) simultaneously using calculator

$a = 5$ (1) $b = 23$ (1)

handwritten method:

(1) $a + 2b = 51$

(2) $a + b = 28$

(2) − (1) $\Rightarrow b = 23$

$a + b = 28 \Rightarrow a = 28 - b$

$= 28 - 23$

$= 5$